Full Paper

DEVELOPMENT OF A FAST AND EFFICIENT TECHNIQUE FOR MODELING BOXED MICROSTRIP STRUCTURE

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ABSTRACT

The inability to derive closed-form analytical solution to Maxwell's equations under various complex interface and boundary conditions is overcome by Computational Numerical Technique. Computational Electromagnetics offers the possibility for modeling to be undertaken within a significantly shorter time than it would be necessary for building and testing the appropriate prototype via experimental procedures, thus reducing the cost associated with the design process. This study describes an accurate and computationally efficient technique for the analysis and design of boxed microstrip circuits. The theoretical derivations are based on the Electric Field Integral Equation (EFIE) that involves the formulation of Green's functions that accurately describe the boxed structure, and rooftop basis functions to approximate the current on the patches. The simulated results compare favourably with experimental results.

Keywords: *Computational Electromagnetics, Boxed microstrip circuits, Electric Field Integral Equation (EFIE), Green's functions, Rooftop basis functions.*

1. INTRODUCTION

Increase in the use of boxed microstrip structures at microwave frequencies has created interest in the study of their dispersion characteristics. The need to simulate microstrip model efficiently has become important because adjustments after fabrication are virtually impossible. A number of different techniques have been employed to demonstrate the dispersion effects of these lines [1]-[5]. Recently, the efficiency of rigorous hybrid technique has been demonstrated [6]. In spite of the accuracies of all these techniques, the challenge to present or demonstrate a fast and more efficient analysis is always still there.

In Method of Moments (MoM) for boxed structures, a major percentage of the computation time is attributed to MoM matrix filling and inversion. Galerkin's MoM in spectral domain has some unique features that accelerates the procedure. It allows one to transform the convolutions into algebraic products, thus avoiding the time consuming numerical evaluation of complicated integrals. This makes the method to be computationally fast and more efficient if basis functions that can be expanded into simple functions that have analytical Fourier transform are equally chosen.

The purpose of this short paper is to develop a model for generating baseline data in the design of more accurate microstrip circuit at microwave frequencies. It also provides the means for reducing the design time and cycle cost of the circuit. The accuracy of the solution is demonstrated by comparing the computed results with those published by other authors.

2. MATHEMATICAL MODEL

A summary of the key steps used in modeling the boxed microstrip structure shown in figure 1, will be outlined here. The structure consists of a conducting strip etched on a dielectric substrate and enclosed in a box of perfectly conducting material. Throughout the analysis, the conducting strip is assumed to have infinitesimal thickness, and the metallic walls of the box and the conducting strip are perfect conductors. It is also assumed that the

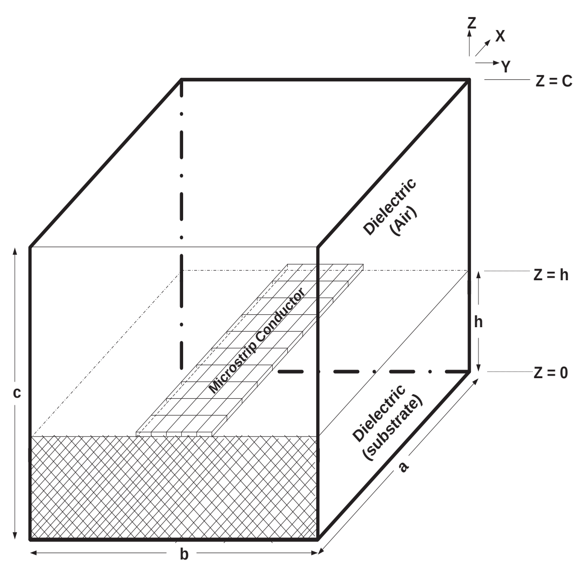

Figure 1: A boxed Microstrip Structure

substrate is lossless and non-magnetic. A time dependence e^{jwt} is used but suppressed to avoid unnecessary complexity (Note: the time-harmonic variation of fields eliminates the time dependence from Maxwell's equations, thereby reducing the space-time dependence to space dependence only [7]).

2.1. Electric Field Integral Equation (EFIE)

The transverse components of the Electric field are expressed in terms of source current density in the box as

$$\vec{E}^{scat}(\vec{r}) = \iint_S \overline{\overline{G}}(\vec{r}/\vec{r}')\cdot\vec{J}(\vec{r}')dS' \quad (1)$$

where $\overline{\overline{G}}(\vec{r}/\vec{r}')$ is the dyadic Green's function, $\vec{J}(\vec{r}')$ is the unknown surface current density, $\vec{r}$ is the position vector of field point (x, y) and $\vec{r}'$ is the position vector of source point (x', y').

The total Electric field in the vicinity of the microstrip conductor can be expressed as the sum of the incident and scattered field component as

$$\vec{E}^{scat.}(\vec{r}) + \vec{E}^{inc.}(\vec{r}) = \vec{E}^{Total}(\vec{r}) \quad (2)$$

In addition, since the conductors are perfect conductors, the total Electric field vanishes along the metallic surface, that is,

$$\iint_S \overline{\overline{G}}(\vec{r}/\vec{r}')\cdot\vec{J}(\vec{r}')\,dS' = -\vec{E}^{inc.}(\vec{r}) \quad (3)$$

An approximation is adopted for the unknown surface current density by meshing the surface of the microstrip conductor into smaller rectangular cells as shown in figure 2, and rooftop basis functions $I_u(r')$ with unknown coefficients γ_u are used for the approximation of the surface current density distribution. Thus, the current density can be written as

$$\vec{J}(\vec{r}') = \sum_{u=1}^{U}\gamma_u I_u(r')\hat{r}' = \sum_i \alpha_i I_{xi}(x',y')\hat{x}' + \sum_j \beta_j I_{yj}(x',y')\hat{y}' \quad (4)$$

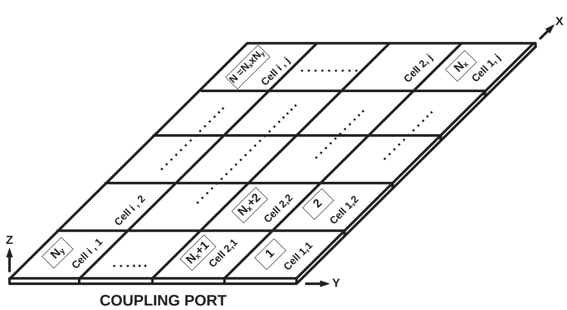

Figure 2: A surface meshed into smaller cells

When assigning basis functions, half-subsectional rooftop basis functions are used for cells representing coupling ports while full-subsectional functions are used on every other cells. To solve for the unknown coefficients α and β, we substitute equation (4) into (3) and apply Galerkin's scheme of MoM to get

$$\left\langle \vec{W}_t(\vec{r}'), \iint_S \overline{\overline{G}}(\vec{r}/\vec{r}')\cdot\vec{J}_u(\vec{r}')dS' \right\rangle = \left\langle \vec{W}_t(\vec{r}'), V_t\delta(\vec{r}-\vec{r}') \right\rangle \quad (5)$$

for t = u = 1, 2,.....U

where, U is the total number of basis functions and $\vec{W}_t(\vec{r}')$ is the weighting functions.

2.2. Spectral Domain MOM

The metallic side walls of the box act like perfect mirrors for the Electromagnetic fields and currents, thus creating a periodic series of images in the lateral directions [8]. The method of moment takes advantage of this lateral periodicity by formulating the problem in spectral domain. The dyadic Green's, basis and weighting functions are subjected in both transverse directions x and y to a discrete double Fourier transform to obtain a set of box modes (in terms of k_x and k_y) that propagate in a direction normal to the material interface. Each of these modes is expressed analytically as a rectangular waveguide mode that resonates between the top and bottom walls of the box [9]. After performing double Fourier transform on equation (5), it becomes

$$\sum_m\sum_n\left[\tilde{W}(k_x,k_y)\cdot\tilde{\overline{\overline{G}}}(k_x,k_y)\cdot\tilde{J}(k_x,k_y)\right] = V_t \quad (6)$$

where ~ indicates the Fourier Transform (FT) of the corresponding vector quantity, m & n are integers denoting the TE and TM modes, $\tilde{\overline{\overline{G}}}(k_x,k_y)$ is the Fourier transform of the dyadic Green's function, $\tilde{W}(k_x,k_y)$ is the Fourier transform of the weighting function (which has the same functional form as the basis function), $\tilde{J}(k_x,k_y)$ is the Fourier transform of the surface current density, $\sum_m\sum_n$ is double infinite summation over m and n, and V_t is the amplitude of the terminal voltage. Equation (6) can be written in matrix form as

$$[Z_{tu}][\gamma_u] = [V_t] \quad (7)$$

where the elements of MoM impedance matrix is given as

$$Z_{tu} = \sum_m\sum_n\left(\tilde{I}_t(k_x,k_y)\cdot\tilde{\overline{\overline{G}}}(k_x,k_y)\cdot\tilde{I}_u(k_x,k_y)\right) \quad (8)$$

and the elements of vector γ_u are α_i and β_j, while vector V_t is either 1 or 0: it is 1 for cells representing the terminal ports and 0 for any other cell on the surface of the conductor.

The Fourier transform of the dyadic Green's function is expressed as

$$\tilde{\overline{\overline{G}}}_{xx}(k_x,k_y) = j\omega\mu A_c(Y_t k_x^2 - Y_s k_y^2)\ ;$$
$$\tilde{\overline{\overline{G}}}_{xy}(k_x,k_y) = \tilde{\overline{\overline{G}}}_{yx}(k_x,k_y) = j\omega\mu A_c(Y_t + Y_s)k_x k_y\ ;$$
$$\tilde{\overline{\overline{G}}}_{yy}(k_x,k_y) = j\omega\mu A_c(Y_t k_y^2 - Y_s k_x^2)\ . \quad (9)$$

where

$$Y_t = \frac{k_z' \tan(k_z' h')}{k_o^2 \left(\dfrac{\varepsilon k_z' \tan(k_z' h')}{k_z \tan(k_z h)} - 1 \right)}$$

$$Y_s = \frac{1}{k_z' \cot(k_z' h') - k_z \cot(k_z h)}$$

$$A_c = \frac{\delta_{mn}}{ab k_c^2}$$

$$\delta_{mn} = \begin{cases} \text{for TE mode}, & \text{for TM mode} \\ 1 \quad 0 & m = n = 0 \\ 2 \quad 0 & m \text{ or } n = 0 \\ 4 \quad 4 & m \cdot n \neq 0 \end{cases}$$

$k_c^2 = k_x^2 + k_y^2$ = cutoff wave number

$$k_x = \frac{m\pi}{a}, \qquad k_y = \frac{n\pi}{b}, \qquad k_z = \sqrt{k_o^2 - k_x^2 - k_y^2},$$

$$k_z' = \sqrt{k_o'^2 - k_x^2 - k_y^2}, \ k_o = \sqrt{\varepsilon} k_o', \ k_0' = \frac{10^{-8}\omega}{2.998}.$$

3. Numerical Model

The matrix elements have been expressed in terms of algebraic product involving the Fourier transform of the dyadic Green's and basis functions. In order to accelerate the computation of the double summation in equation (8), the frequency independent components of the FT of dyadic Green's function were introduced. These components are obtained from equation (9) by computing the asymptotic values of k_z as the integers m and n tend to infinity.

$$\tilde{\tilde{G}}_{xx}(k_x, k_y) = j\omega\mu A_c \left(\frac{k_y^2}{2k_c} - \frac{k_x^2 k_c}{k_o^2(\varepsilon+1)} \right),$$

$$\tilde{\tilde{G}}_{xy}(k_x, k_y) = \tilde{\tilde{G}}_{yx}(k_x, k_y) = -j\omega\mu A_c \left(\frac{1}{2k_c} + \frac{k_c}{k_o^2(\varepsilon+1)} \right) k_x k_y$$

$$\tilde{\tilde{G}}_{yy}(k_x, k_y) = j\omega\mu A_c \left(\frac{k_x^2}{2k_c} - \frac{k_y^2 k_c}{k_o^2(\varepsilon+1)} \right). \qquad (10)$$

The number of basis functions was determined by calculating the guide wavelength at the highest frequency of interest and the rule of thumb for cells per guide wavelength was adopted [10]. Figure 3 shows a one-port circuit with substrate relative permittivity ε_r of 2.2, substrate thickness h of 6.35mm and box height c of 21.79mm.

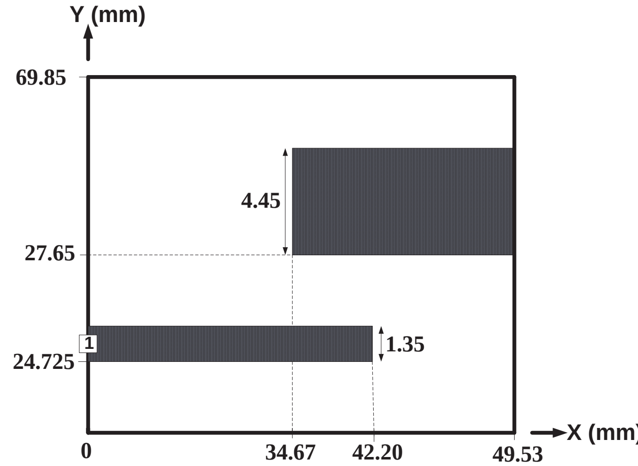

Figure3: Top view of one port boxed microstrip circuit [5]

Using 40 basis functions, the simulated result as shown in figure 4 shows good agreement with the experimental result reported in [5]. To further demonstrate the efficiency of the analysis, figure 5 with box height c = 16mm, substrate thickness h = 1.272mm and relative permittivity of 10 was analyzed. Figure 6 shows the plot of transmission coefficient of the simulated result along side with the published result.

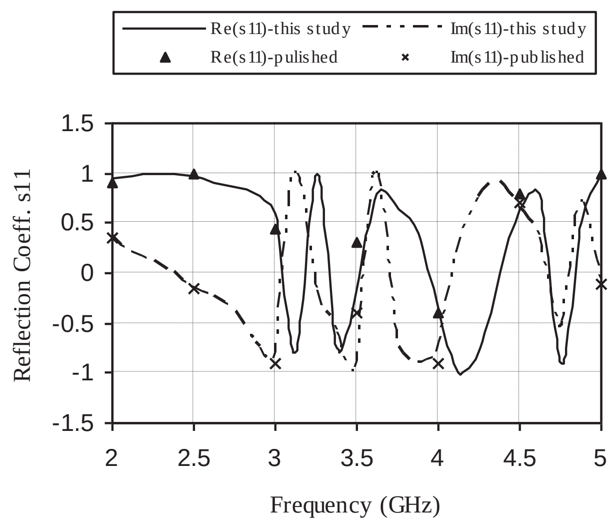

Figure 4: Plot of Reflection Coefficient versus Frequency

The computation is extended to circuit with many microstrip patches. Figure 8 compares the simulated and experimental results [4] of the microstrip circuit shown in figure 7.

4. Conclusion

In this analysis, a Method of Moments formulation has been applied using rooftop basis functions on uniformly meshed rectangular cells and a delta-gap excitation model. The resulting semi-analytical (closed-form) expressions for spectral dyadic Green's functions enhanced the extraction of frequency independent

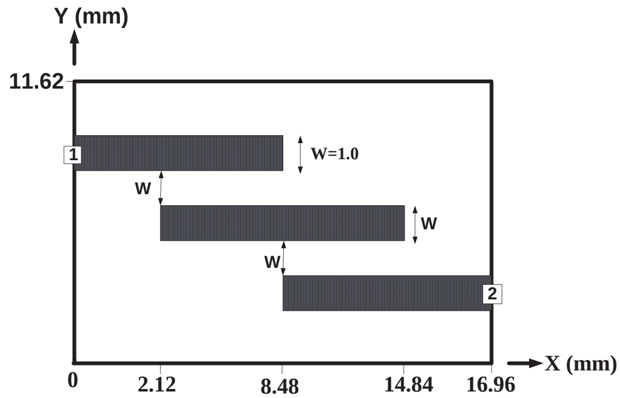

Figure5: Top view of two port boxed microstrip circuit [5]

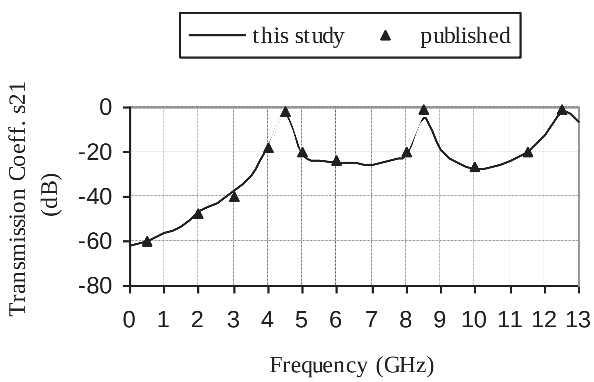

Figure 6: Plot of Transmission Coefficient versus Frequency

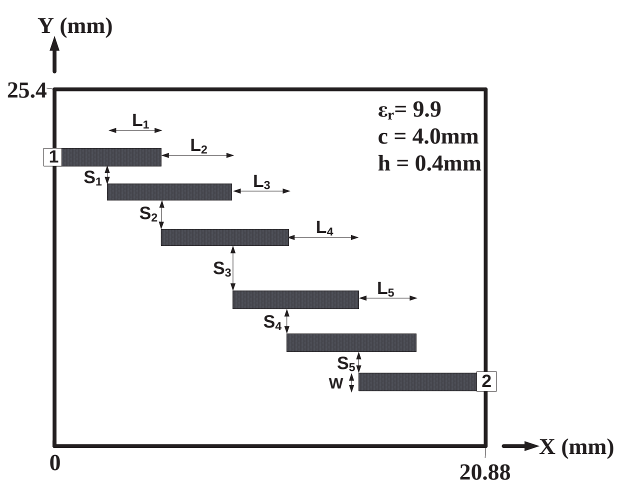

Figure 7: Top view of two port boxed microstrip circuit[4]

components of the reaction equation which reduced the recurrent calculation to fewer terms, thus reducing the required CPU memory and computational time. The simulated results obtained compare favourably with experimental results obtained from literature as well as simulated results obtained elsewhere. This good agreement indicates that the method can be applied to more complex structures with known Green's functions, especially layered structures in microwave integrated circuits.

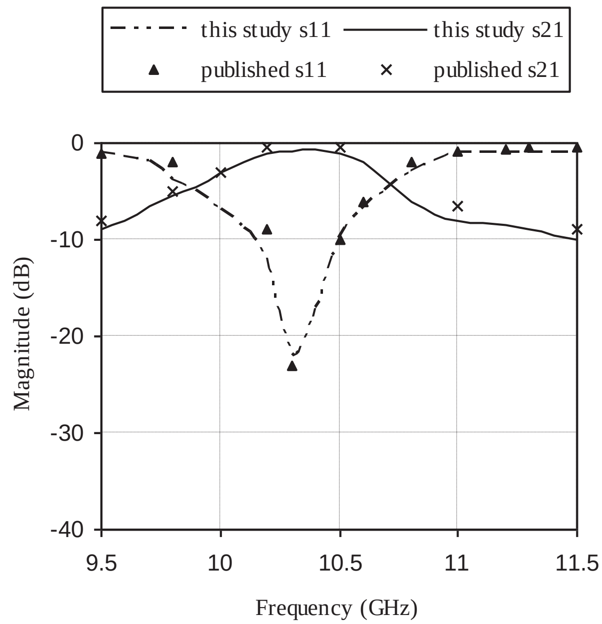

Figure 8: Plot of scattering parameters versus frequency

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